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ELECTRICAL ANALOGUE MODEL FOR MOVING STRIPE SOURCES OF FRICTIONAL HEAT ON SURFACES OF SOLID BODIES

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Abstract — Our aim is to determine the temperature field of a plane body. The surface of the body is heated up by moving sources of frictional heat. For this purpose we use an electrical analogue model from which we take measurements and compare them with theoretically determined figures by Carslaw and Jaeger [2]. Within the bounds of measuring accuracy they correspond with each other. The structure of the used model and electronics are described. Temperatures directly near the surface (micrometer region) can be given for friction loaded solid bodies. Finally, it is shown how the electrical analogue method can also be used for the determination of temperatures in materials, whose surface is disturbed by cracks and where only difficult computation methods can be used or computations are impossible.

NOMENCLATURE

A,	area;
a,	thermal diffusivity;
C,	capacitance;
c,	specific heat;
I,	non-dimensional temperature, equation (3);
$K_0(u)$,	Bessel function;
L,	non-dimensional speed;
l,	half breadth of stripe sources;
P^*,	surface power [W/m^2];
Pe,	Peclet number;
q,	strength of line straped heat source [W/m];
t,	time;
t_M,	model time;
U,	voltage;
u,	parameter;
v,	speed;
X,	non-dimensional space coordinate;
x,	space coordinate;
x_M,	model dimension;
x',	integration variable;
Z,	non-dimensional space coordinate;
z,	space coordinate;
z_M,	model dimension.

Greek symbols

γ,	capacitance per unit area [F/m^2];
ΔI,	current;
Δl_M,	breadth of a boundary electrode;
ϑ,	temperature;
κ^*,	surface conductivity [1/Ω];
λ,	thermal conductivity;
ρ,	density.

1. INTRODUCTION

FRICTION occurs on the contact of two contacting solid bodies. The real contact surface consists of many real contact points, which are irregularly distributed on the frictional surface of nominal contact.

The size and dimensions of these points depend on the microgeometry of the frictional surface, the load and the mechanical material qualities of the contacting bodies [1].

Frictional heat is produced at the real contact points, which move over the nominal contact during the friction process. That is why frictional load causes unsteady temperature fields near the surface of the solid bodies.

As a simple model for moving sources of frictional heat on the surface of solid bodies we choose moving stripe sources. The resulting two-dimensional unsteady temperature field in the solid body is mathematically described by equations, which are evaluated by numerical methods with the help of digital computers [2–4].

Another method to solve this problem is the electrical analogue model, which makes a quick solution of difficult examples possible. We aim to demonstrate this in our paper.

2. MATHEMATICAL BASIS

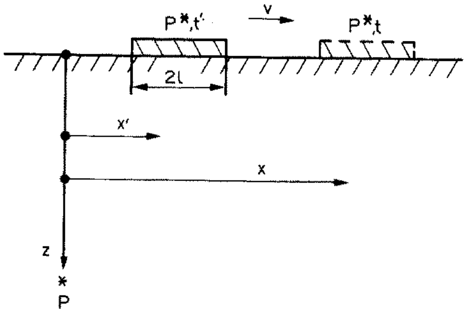

FIG. 1. Coordinate system.

A stripe source with the breadth $2l$ ($-l \leq x' \leq +l$)

moves in positive x-direction on the surface of a solid body, which is supposed to be of infinite extent (semi-infinite space, see Fig. 1). We suppose that the coordinate system is fixed to the solid body. The speed of the stripe source is v, the movement has been going on long enough and the stripe source has the surface power $P^* = (\mathrm{d}q/\mathrm{d}x')$. Then follows, according to [2] for the temperature ϑ in the depth z and at the place x of the solid body:

$$\vartheta = \frac{P^*}{\pi \cdot \lambda} \int_{-l}^{+l} \exp\left\{\frac{v(x-x')}{2a}\right\} \cdot K_0\left\{\frac{v}{2a}\sqrt{[(x-x')^2 + z^2]}\right\} \mathrm{d}x'. \quad (1)$$

The integral is simplified by the following substitutions:

$$X = \frac{vx}{2a}; \quad Z = \frac{vz}{2a}; \quad L = \frac{vl}{2a} = \frac{Pe}{4};$$

$$u = \frac{v(x-x')}{2a}.$$

Then results:

$$\vartheta = \frac{2P^* \cdot a}{\pi \cdot \lambda \cdot v} \int_{X-L}^{X+L} e^u \cdot K_0[\sqrt{(u^2 + Z^2)}]\,\mathrm{d}u. \quad (2)$$

A solution of the equation (2) cannot be given in a closed form. A numerical integration, which can be carried out with the help of the Romberg-method and a digital computer, becomes necessary [4].

3. ELECTRICAL ANALOGUE MODEL

The electrical analogue model is clear and very variable. The results are exact enough. This model is based on the same mathematical structure of the differential equations of the original (temperature field) and of the model (electric flow field). By dimensionless writing of the differential equations we can derive the dimensioning equations of the used model [5]. The analogous physical quantities are given in Table 1.

The construction of the model becomes especially simple if a continuous (thick layer-) RC-model is used. The continuous model has a sandwich structure [6] and consists of an electric resistive film (electrically conductive paper), which is on an insulating layer (self-adhesive thin-sheet plastic) as a dielectric. The second plate side of the capacitor, a thin-sheet copper, is formed under the dielectric. Since the dielectric layer is relatively thick (approx. 100 μm), very small capacities occur per unit of area and thus the measurement requirements to the used electronics become higher (very short model times in the quantity of μs). The model receives a current through boundary electrodes of the electrically conductive paper. These electrodes are covered with conductive silver. Various boundary conditions are realised with a corresponding arrangement of the boundary electrodes and current adjustment.

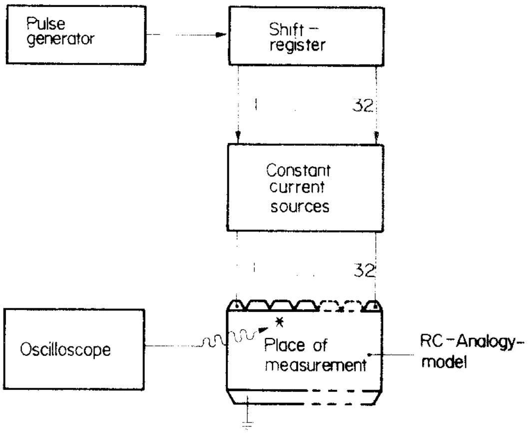

FIG. 2. General wiring diagram.

The temperature field is found with the help of oscilloscope voltage measurements at the model and conversion with the temperature–voltage scale. The boundary electrodes of the model for the imitation of moving stripe sources must receive defined current impulses for this purpose.

4. ELECTRONIC CONSTRUCTION

In order to produce defined current impulses, which imitate the moving stripe sources on the surface of the solid body, rectangular impulses of constant current must be connected. This is possible with the help of

Table 1. Comparison between the parameter of the original and the model

Original		Model	
Temperature	ϑ	Voltage	U
Time	t	model time	t_M
Dimension	x, z	model dimension	x_M, z_M
Thermal conductivity	λ	surface conductivity	κ^*
Thermal capacity per unit volume	$\rho \cdot c$	capacitance per unit surface	$\gamma = \frac{\Delta C}{\Delta A_M}$
Surface power	P^*	current per boundary electrode	$\frac{\Delta I}{\Delta l_M}$

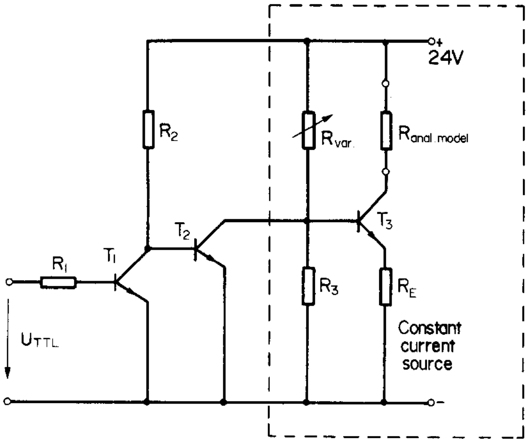

FIG. 3. Switchable constant current source.

switchable sources of constant current and a central control unit on the basis of digital integrated switching circuits. Figure 2 shows the general wiring-diagram. The main part of the control unit is a shift register. On these (in this case 32) outputs of the shift register the potential 'High' occurs successively in the breadth of the clock frequency. This potential controls the sources of constant power through a transistor switching element. A constant current through the model can be adjusted with a potentiometer in the base voltage divider (see Fig. 3). In this way rectangular impulses of constant current can be produced up to a quantity of maximum 1 MHz.

5. RESULTS

At first a moving stripe source was imitated on a surface of a solid body. Figure 4 shows the comparison between the calculated and the measured results. Here the non-dimensional temperature is used as ordinate

$$I = \int_{X-L}^{X+L} e^{u} \cdot K_0[\sqrt{(u^2 + Z^2)}]\,du. \qquad (3)$$

This case is valid for the depth $Z = 0.2$ and the non-dimensional speeds $L = 0.2$ and 5.

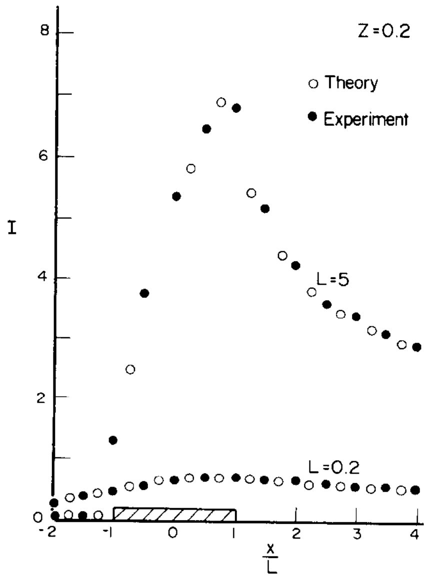

FIG. 4. Comparison between experiment and theory.

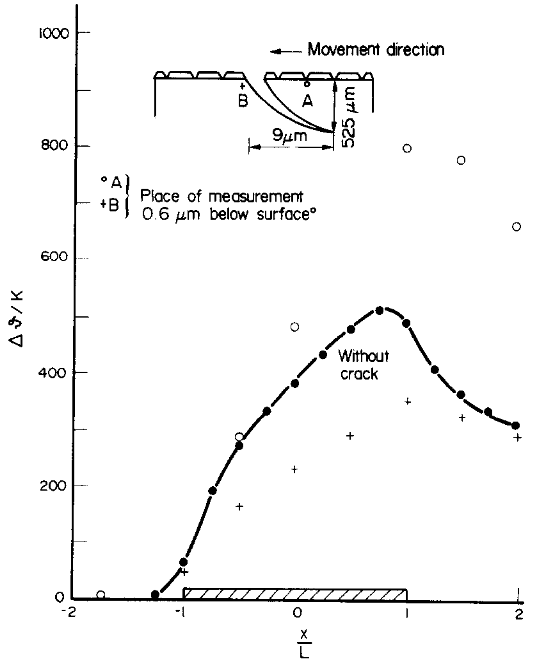

FIG. 5. Measurement with crack.

To make it clear the source was sectioned. The comparison between computation and measurement shows that the results correspond to each other. This shows that the electrical analogue model can be used for experiments with moving sources of frictional heat on surfaces of solid bodies. Thus it is possible to use the model for difficult or incalculable cases. A first application is shown on Fig. 5. The surface of a solid body has a crack below 45°, parallel to the direction of movement [7]. The breadth of the friction source is supposed to be 24 μm. Figure 5 demonstrates that the temperature between crack and surface becomes much higher, while it is much lower behind the crack (looking into the direction of the source) than without the crack. That is why the temperature gradient at the previously disturbed place becomes much higher and leads to higher thermal stresses.

In future we intend to use the model for experiments with finite dimensions of solid bodies, rough surfaces and material inhomogeneities.

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MODELE ANALOGIQUE ELECTRIQUE POUR DES SOURCES MOBILES EN BANDE DE CHALEUR PAR FROTTEMENT SUR LES SURFACES DE CORPS SOLIDES

Résumé—Il s'agit de déterminer le champ de température d'un corps plan. La surface du corps est chauffée par des sources mobiles de chaleur par frottement.

On utilise un modèle analogique électrique sur lequel on effectue les mesures et on compare avec les résultats théoriques de Carslaw/Jaeger. Dans les limitations de la précision de mesure il y a une bonne correspondance. On décrit les structures de modèle utilisé et l'électronique. Pour des corps solides en frottement, on peut donner les températures directement près de la surface (région micrométrique).

On montre finalement comment la méthode analogique électrique peut aussi être utilisée pour la détermination des températures dans les matériaux dont les surfaces sont perturbées par des craquelures et où les méthodes de calcul seules utilisables sont difficiles ou inexploitables.

ELEKTROANALOGE MODELLNACHBILDUNG BEWEGTER STREIFENFÖRMIGER REIBUNGSWÄRMEQUELLEN AN FESTKÖRPEROBERFLÄCHEN

Zusammenfassung — Es soll das Temperaturfeld in einem ebenen Körper ermittelt werden, dessen Oberfläche sich durch bewegte Reibungswärmequellen erwärmt. Dazu wird ein Elektroanalogiemodell benutzt, an dem Messungen durchgeführt und mit theoretisch nach Carslaw und Jaeger [2] berechneten Werten verglichen werden. Im Rahmen der Meßgenauigkeit besteht zwischen beiden Übereinstimmung. Aufbau des verwendeten Modells und der benutzten Elektronik werden beschrieben. Es lassen sich Temperaturen für reibungsbeanspruchte Festkörper in unmittelbarer Oberflächennähe (Mikrometerbereich) angeben. Abschließend wird gezeigt, wie die Methode der Elektroanalogie auch bei der Temperaturermittlung in durch Risse gestörten Werkstoffbereichen eingesetzt werden kann, wo Berechnungsverfahren nur mit sehr großem Aufwand oder gar nicht anwendbar wären.

ЭЛЕКТРИЧЕСКАЯ АНАЛОГОВАЯ МОДЕЛЬ ПЕРЕМЕЩАЮЩИХСЯ ЛЕНТОЧНЫХ ИСТОЧНИКОВ ТЕПЛОТЫ ТРЕНИЯ НА ПОВЕРХНОСТЯХ ТВЕРДЫХ ТЕЛ

Аннотация — Исследование предпринято с целью определения температурного поля, возникающего при нагревании плоского тела движущимися источниками джоулева тепла. Для решения этой задачи использована электрическая аналоговая модель. Проведенное сравнение полученных результатов с результатами теоретического расчета по методу Карслоу–Егера показывает хорошее совпадение в пределах погрешности измерений. Дано описание модели и электронной схемы. Температура твердых тел при наличии трения может измеряться непосредственно у поверхности (в микронном диапазоне). Показано также, каким образом электрический аналоговый метод может использоваться для определения температуры материалов с трещинами на поверхности, когда численные расчеты весьма затруднены или полностью невозможны.